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Smart Grid PMU Allocation Using Genetic Algorithm

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***Abstract* — A genetic algorithm (GA) based approach for reliability placement of phasor measurement units (PMUs) in smart grid is proposed. The algorithm combines two conflicting objectives which are maximization of the reliability of observability and minimization of the number of PMU placements for ensuring full system observability. The multi-objective problem is formulated as a nonlinear optimization problem and genetic algorithm approach is employed for solving the large scale bus systems. The optimization model is solved for IEEE 14, 30, 57, 118, and 2383 standard bus systems. The effectiveness of the proposed approach has been demonstrated by comparing results with exact algorithms for smaller problem sizes. The results suggest that by employing genetic algorithm, the system reliability of observability is improved by approximately 48% as compared to traditional optimal PMU placement. According to results, the proposed approach achieve significant cost savings (~17%-~50%) compared to available reliability based models in literature.**

Keywords-Genetic algorithm (GA); multi-objective optimization; nonlinear programming; phasor measurement units (PMUs); reliability modeling

I. Introduction

The Wide Area Measurement System (WAMS) has become the most important part of the electrical power network monitoring, control and security nowadays [1]. Secure monitoring and control of the smart grid has been significant since failures such as loss of generation, transmission line outages or metering failures could turn in to blackouts and cascade tripping. Reliability and robustness of the WAMS is highly crucial since the security of the smart grid highly depends on it where a reliable WAMS can prevent smart grid from turning into cascade tripping and blackouts. Phasor Measurement Units as state estimator devices provide Global positioning System (GPS) based time synchronized measurement of current and voltage phasors in power network [1]. This feature of time stamped state estimation via signals from GPS satellite transmission makes the PMUs as the key elements of WAMS [2, 3].

A bus is an electrical conductor, which serves as a conducting pathway for continuous connection of the loads and the sources of electric power between different parts of a power grid. Transmission between buses is made through lines in the network. A bus is called observable when the voltage phasor at that bus is estimated and the power system is called to be observable if the measurement sets and their distributions are sufficient for solving the current state. PMU placement at a given bus potentially allows measurement of the voltage phasor at that bus and current phasors of all lines that are incident to the bus. Availability of the voltage phasor of a bus and the entire incident line currents, the voltage phasor at adjacent buses can be calculated using the Ohm's law. Therefore, the presence of a PMU on a bus makes that particular bus and all of its immediate neighboring busses observable [4, 5]. The use of PMUs at each bus leads to a simplified linear state estimator [6]. Hence, addition of the PMU at all strategic buses in the power network allows direct measurement of the state of the network [4, 5]. Therefore a network can potentially be observable with a lesser number of PMUs than the number of buses. As a result, the PMU placement problem concerns with achieving the entire network observability with the minimum possible cost through placement of the minimum number of PMUs on strategic locations.

Several algorithms for PMU placement problem developed in the literature considering full network observability. A generalized integer linear programming for optimal PMU placement problem to address the cases of redundancy, complete and partial observability and pre-existing conventional measurements considering zero injection buses is developed in [7]. A joint placement of PMUs and conventional flow measurements model for full observability of power systems is developed in [8].

Reference [9] show that the PMU placement is Non-deterministic Polynomial time hard (NP complete) problem. They present a new simpler definition of graph observability. Several heuristics are developed for optimal placement of PMUs to ensure observability for solving large scale problems. Reference [10] proposed a non-dominated sorting genetic algorithm for the PMU placement problem. To reduce the initial number of PMU's candidate locations, they considered the conflicting objectives of minimization the number of PMUs and maximization of the measurement redundancy by estimating the individual optimal solution for these conflicting objectives using the graph theoretical procedure and a simple genetic algorithm. Then using the non-dominated genetic algorithm they searched for the best tradeoff. Reference [11] investigated the application of immunity genetic algorithm for the optimal PMU placement problem.

Reference [12], formulated the optimal PMU placement problem as a quadratic minimization problem with continuous decision variables subject to nonlinear observability constraints. Reference [13], proposed an analytic technique for optimal PMU placement problem considering both long-term economic aspects and existing technical issues. An information-theoretic approach for PMU placement problem presented in [14]. They proposed mutual information criterion between the PMU measurements and the power system states to be able to facilitate robust PMU placement by explicitly modeling probabilistic PMU outages. Reference [15], developed a systematic approach to minimize the required number of PMUs while guarantying robustness of the least absolute value state estimation.

System reliability is the probability that a system will perform its intended function for a given period of time under pre specified operating conditions [16]. For a system to perform its intended functions, it is important that all components and sub-systems are highly reliable and able to perform specified functions within the given requirements. The reliability of data transmission in power systems highly depends on the reliability of PMUs covering each bus since PMUs are key component of WAMS. Therefore it is crucial to compute and consider the reliability of the arrangement, given a placement that is optimal with respect to cost.

The timely detection of the possibility of blackouts is crucial to allow effective control and protection of the power system. It is intuitively clear that protecting against loss of observability under failures (such as transmission line faults, bus faults, outages, or metering failures) requires a level of redundancy with additional PMUs. Nowadays with the extensive use and installment of PMUs in power systems, costs for both PMU and installments are decreasing. The need for secure and robust WAMS and decreasing overall PMU placement costs will allow considering placement of the redundant number of PMUs with the goal of reaching a specific reliability of observability. Reference [16] developed a two- stage optimization model for the optimal PMU placement problem from a reliability standpoint, where redundancy levels for all buses in the system are assumed to be identical. Such an optimization task involves two conflicting objectives, cost minimization and reliability maximization.

However, the assumption of identical redundancy levels (i.e., bus reliabilities (r)) at all buses in the system makes the solution space smaller thus resulting in infeasible solutions in many cases. A nonlinear multi objective optimization approach has been used in [17] to improve the feasibility of the solution. However, the addition of the reliability maximization to the objective function makes the NP-complete optimal PMU problem even more complex and renders it unsolvable for large scale problems by exact solution approaches.

In practice the resources could be limited because of the high price of purchasing and installing the PMUs. In this case, the decision maker will decide to allocate the limited recourses either to the strategic locations or to cover the maximum possible buses. Therefore the PMU placement problem from a maximum covering standpoint was considered in [18]. In the proposed model, the number of existing PMUs is factored as inputs into the model. The maximum coverage thus dictated by this input is subject to the system topology.

In [19], we present a preliminary conference version of the application of the genetic algorithm on IEEE 14 and 30 bus systems to test the efficiency of the approach on the reliability based PMU placement. In this paper, we increase the scope, by developing a revised genetic algorithm, and test the approach on large problem instances, namely IEEE 57, 118, and 2383 test systems. The model not only enables solution of the large scale 2383 bus system but also finds better tradeoffs compared to the previous approaches developed in the literature. The main goals are ensuring full system observability while aiming for a pre-specified level of reliability of observability and minimizing the cost of placing PMUs. The weights associated to the goals derived both from the relative importance given to the goals and based on simulation results. The need to reach pre specified overall system reliability of observability dictates the placement of additional PMUs as compared to traditional PMU placement problems and eventually results in more expensive solution. This extra cost is the cost of achieving a higher reliability level.

The paper is organized as follows. Section II describes the Multi-objective problem formulation and section III introduces the genetic algorithm approach. In section IV, we discuss the implementation of genetic algorithm on the multi-objective PMU placement problem and discuss the results on the IEEE test cases. Section V concludes the paper.

II. Multi-Objective formulation

The improved reliability based PMU placement briefly will be presented in this section. Detailed explanations on basic reliability based PMU placement can be found in [16]. The assumption of identical redundancy levels (i.e., bus reliabilities (r)) at all buses in the system presented in [16] makes the solution space smaller resulting in infeasible solutions for many cases. The improved reliability model compatible with Multi-objective optimization which relaxes the assumption of identical redundancy levels (bus reliabilities (r)) at all buses in the system will be described.

PMU placed at a given bus measures both the voltage Phasor of that bus and the current phasors of adjacent lines. Then providing the availability of accurate branch models and by using the Ohm's law the voltage phasors of adjacent buses can be obtained. Therefore placement of a PMU at a given bus allows direct measurement of voltage Phasor at that bus and computation of the voltage phasors at immediate neighboring buses. Thus, the entire system will be observable if all buses are covered with one or more PMUs. Therefore, buses are connected in series from a reliability point of view. In case none of the PMUs are redundant, the failure of any PMU would result in the system failure. Thus observability of the entire system is given as:

$$A = \bigcap_{i=1}^{n} A_i \tag{1}$$

where A represents the observability of the entire system, A_i represents the observability of the i^{th} bus and n is the number of the buses in the system. Using the concept of union and intersection from set theory yields:

$$ROB = P(A) = \prod_{i=1}^{n} P(A_i) = \prod_{i=1}^{n} r_i \tag{2}$$

where r_i represents the reliability of observability level for i^{th} bus and *ROB* is the overall system reliability of observability. It should be considered that if bus i is covered by more than one PMU, the redundant PMUs will be connected in parallel from the reliability standpoint. Therefore, if all PMUs covering i^{th} bus fail simultaneously, observability of i^{th} bus will fail. Assuming identical reliability for PMUs, we can define reliability of observability of bus i (r_i) as:

$$r_i = 1 - \prod_{j=1}^{f_i} q_j = 1 - q_j^{\,f_i} \tag{3}$$

where q_j represents the probability of failure of j^{th} PMU and f_i is the total number of PMUs covering i^{th} bus.

Hence $\prod_{j=1}^{f_i} q_j$ denotes probability of failure of all PMUs observing i[th] bus. Substituting (3) into (2)

$$ROB = \prod_{i=1}^{n} (1 - q_j^{\,f_i}) \tag{4}$$

This reliability index is included into placement constraints as follows. Defining X as a binary decision variable vector defined by:

$$x_i = \begin{cases} 1 \text{ if a PMU is present at bus } i \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

the binary connection matrix A of the system can be directly obtained by transforming the bus admittance matrix's entries into binary form defined by:

$$A_{i,j} = \begin{cases} 1 & \text{if either } i = j \text{ or } i \text{ is adjacent to } j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

By definitions of x_j and $A_{i,j}$, the total number of PMUs covering i^{th} bus(f_i) can be developed mathematically:

$$f_i = \sum_{j=1}^{n} A_{i,j} x_j \tag{7}$$

F_i's for the IEEE 14 standard bus system in Fig.1are as follows:

$$f_1 = x_1 + x_2 + x_5$$

$$f_2 = x_1 + x_2 + x_3 + x_4 + x_5$$

$$f_3 = x_2 + x_3 + x_4$$

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9$$

$$f_5 = x_1 + x_2 + x_4 + x_5$$

$$f_6 = x_6 + x_{11} + x_{12} + x_{13}$$

$$f_7 = x_4 + x_7 + x_8 + x_9$$

$$f_8 = x_7 + x_8$$

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14}$$

$$f_{10} = x_9 + x_{10} + x_{11}$$

$$f_{11} = x_6 + x_{10} + x_{11}$$

$$f_{12} = x_6 + x_{12} + x_{13}$$

$$f_{13} = x_6 + x_{12} + x_{13} + x_{14}$$

$$f_{14} = x_9 + x_{13} + x_{14}$$

Hence the Multi-objective PMU placement model has been formulated with two main objectives. The first main objective is the maximization of the reliability of observability of the system with the minimum desired/predefined reliability of observability of R_{min}. The second main objective is the minimization of the number of PMUs. The PMUs assumed to be identical; therefore minimizing the number of PMUs will result in cost minimization. The developed genetic algorithm model will also maintain the full system observability, since based on the reliability of observability definition; reliability of observability of the partial observable system will be zero.

Number of PMUs must increase to reach higher redundancy in observability. Therefore, the objectives of PMU minimization and reliability of observability maximization are in conflict. To resolve the conflict, relative weights should be assigned to each objective to combine two conflicting objective functions into a single objective function .The Multi-objective programming model is as follows:

$$Max\ w_1\left(ROB - R_{\min}\right) + w_2\left(\sum_{i=1}^{n} x_i\right)^{(-1)} \qquad (8)$$

where f_i is given in (7), R_{min} is the desired system wide reliability level, $\sum_{i=1}^{n} x_i$ is the total number of buses to be placed in the system and w_i defines the weight associated to each objective and sum of w_i is equal to one . w defines the weight associated with the objectives and is a decision tool for the problem solver. If reliability maximization is more important, then w_1 should be increased. However if cost is more important than reliability, then smaller value of w_2 should be used. Therefore since the objective in this paper is to reach the predefined system reliability of observability

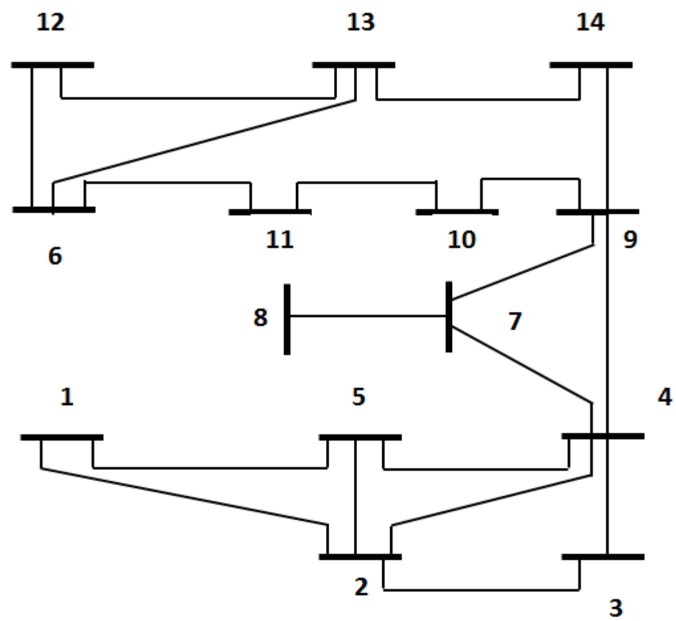

Fig. 1. IEEE 14 Bus system.

with minimum cost and highest possible system reliability of observability; The weights in (8) derived both from the relative importance given to the objectives and based on simulation results. Since the scales are different for R and x, the model in (8) needs to be normalized.

III. Genetic Algorithm Formulation

In the Multi-objective PMU placement problem, a genetic algorithm based approach is utilized. Genetic algorithm is the search heuristic that mimics the process of a natural evolution in which the technique such as the inheritance, mutation, selection, and the crossover operators are being used.

In the solution approach, we have implemented a genetic algorithm based on binary encoding. If the PMU is placed on that particular bus, then the representative value at that particular bus takes the value of 1, and it takes the value of 0 if otherwise. The scheme can be represented by the following operation in a 10-node system

1 0 1 0 1 0 0 0 1 0

Based on this particular representation, the PMUs are placed on the nodes 1, 3, 5, and 9.

For each population, initial population is generated by creating a random number that is uniformly distributed between 0 and bus size. A corresponding threshold value is calculated by dividing that number to the total number of bus size. If the generated random number is smaller than the corresponding threshold value, then in the corresponding bus, PMU is placed; otherwise no PMU is replaced on the position. After the initial population generation, a corresponding fitness function is calculated. The fitness function is calculated based on

- Number of covered buses divided by total number of buses (i.e., θ_1)
- Number of buses that no PMU is placed divided by total number of buses (i.e., θ_2)
- Whether the system threshold reliability level is exceeded or not (i.e., θ_3 -a binary measure)
- Overall system reliability level (θ_4)

The fitness function then can be calculated as

$$\mu = \sum_{i=1} \omega_i \theta_i \tag{9}$$

Where μ is the corresponding fitness function value of the chromosome and ω_i is the corresponding weights associated with the criteria that are listed above.

After the corresponding solutions are ranked according to the descending order of fitness function, based on the roulette wheel selection scheme, the chromosomes that will undergo crossover to produce offsprings are selected. Based on this scheme, the solution which has a higher fitness solution is likely to be selected for producing offsprings for undergoing crossover operator. The roulette-wheel selection is based on the idea that better solutions when go under cross-over operator provide better offsprings. For producing offspring, two different approaches are followed depending on the length of the chromosome (i.e., number of buses). For 14, 30, and 57-bus systems, a traditional two point crossover operator is applied. For the 118 and 2383 bus systems, four point traditional crossover operator is utilized.

In a two-point traditional crossover operator, two crossover sites are randomly selected and the part of the chromosome between those sites is exchanged among theparents. An example of the traditional two-point crossover can be provided as follows:

Parent 1: 1 0 0 0 | 1 0 0 | 0 0 0 1 0 0 0

Parent 2: 0 0 1 1 | 0 1 0 | 1 0 1 0 1 0 1

Offspring 1: 1 0 0 0 | 0 1 0 | 0 0 0 1 0 0 0

Offspring 2: 0 0 1 1 | 1 0 0 | 1 0 1 0 1 0 1

For 118 and 2383 bus systems, four point crossover-operator is applied. In the four point cross-over operator, four crossover sites are randomly selected. In that scheme, the bits between the first and the second, and the third and fourth sites are exchanged among the parents to produce offsprings. In addition to two point crossover-operator [20], the efficiency of multi-point crossover operator especially for the chromosome representations involving long strings has been analyzed in the literature as well [21]. An example of the four-point crossover is presented as follows:

Parent 1: 1 0 0 | 0 1 0 0 | 0 0 0 | 1 0 0 0

Parent 2: 0 0 1 | 1 0 1 0 | 1 0 1 | 0 1 0 1

Offspring 1: 1 0 0 | 1 0 1 0 | 0 0 0 | 0 1 0 1

Offspring 2: 0 0 1 | 0 1 0 0 | 1 0 1 | 1 0 0 0

After offsprings are created, the mutation operator is performed on a bit by bit basis. A random number uniformly distributed between 0 and 1 is generated for each bus in the chromosome representation. If the generated number is smaller than the mutation probability, then the corresponding bit is changed from 0 to 1 or 1 to 0, thus placing or removing the PMU on the corresponding bus.

After all the offpsprings are created, the existing population and created offsprings are ranked based on the descending order of the fitness function which is presented in (9), and a combination of the elitist selection and roulette wheel selection is performed to select the new generation. Again a distinction is made based on the problem size. For 14, 30, 57, and 118 bus systems, top10 chromosomes are selected and included directly in the new generation using the elitist generation scheme. For the 2383 bus system, this number is set to be 50. The remaining chromosomes are selected based on the roulette wheel selection rule. After forming the new generation, the same sequence of procedures are applied (i.e., selection for producing offsprings, crossover, mutation, and the selection for the new generation) on the new generation, and this is repeated until generation limit is reached (i.e., 15,000 for 2383 bus systems, 5000 for the rest).

The genetic algorithm stops after a predefined limit on the generation number is reached. Table I provides the parameters associated with the genetic algorithm. The suggested evolutionary approach based on genetic algorithm is coded in Mathworks Matlab platform without using Genetic Algorithm toolbox. Creating the code helps us tailor

TABLE I
GENETIC ALGORITHM PARAMETERS

Parameter	Value
Population Size	60|500
Number of offsprings created in each generation	30|74
Number of population members selected by the elitist selection rule	10|50
Number of population members selected by the roulette wheel selection	50|450
Mutation probability	0.01
Generation Limit	5000|15000
ω1	4/9
ω2	1/10
ω3	2/5
ω4	1/18

and fine-tune genetic algorithm according to the nature of the problem.

IV. RESULTS AND DISCUSSIONS

The proposed genetic algorithm Multi-objective PMU placement model is implemented for the IEEE 14, 30, 57, 118 and 2383 bus standard test systems. The computations were performed with Matlab2010a. on a 2.66 GHz Intel(R) Core™ 2 Quad CPU with system memory of 2.96 GB. Results are reported with R_{min} = 0.90 for all system types.

A summary of results for the all standard IEEE types are presented in Tables II, III and IV for PMU reliability of 0.95, 0.99, and 0.99833 respectively. The IEEE 2383 test system is missing in Table II since the test system is not able to reach the desired reliability of observability of 0.90 with the PMU reliability of 0.95. In these tables, the required number of PMUs and achieved actual overall system reliability are calculated for PMU reliabilities of 95%, 99% and 99.833% since in reality PMU reliabilities are near 99%.

The PMU locations for the standard IEEE test systems for PMU reliability of 0.95, 0.99 and 0.99833 are shown in

TABLE II
PLACEMENT RESULTS FOR PMU RELIABILITY OF 0.95

IEEE System	#PMU	Reliability Achieved
14	8	0.9329
30	20	0.9142
57	35	0.901
118	82	0.9009

TABLE III
PLACEMENT RESULTS FOR PMU RELIABILITY OF 0.99

IEEE System	#PMU	Reliability Achieved
14	5	0.9315
30	13	0.9123
57	27	0.9004
118	59	0.907
2383	2250	0.9003

TABLE IV
PLACEMENT RESULTS FOR PMU RELIABILITY OF 0.99883

IEEE System	#PMU	Reliability Achieved
14	4	0.9818
30	10	0.9736
57	17	0.9244
118	35	0.9045
2383	1993	0.9004

Tables V, VI and VII respectively. It should be noted that for the IEEE 2383 test system, non-PMU buses rather than PMU-buses are presented for the purpose of brevity.

Based on the results shown in Table III and IV, it is clear that with the increase in the system size, higher redundancy level in terms of the number of PMUs is required to maintain the desired reliability levels.

The usefulness of the genetic algorithm approach further investigated, by comparing the results to the reliability based placement (RBP) results in [16], goal programming based (GPB) approach in [22] and optimal PMU placement (OPP) results in [8]. Results for the case of p=0.99 with minimum desired system wide reliability of observability level of R_{min}=0.90 are shown in Table VIII for IEEE 14, 30, 57, 118 and 2383 standard bus systems. The genetic algorithm approach is shown to be efficient as compared to other approaches presented in [8, 16 and 22].

In literature, the approaches presented in the previous paragraph fail to solve IEEE 2383 bus system with reliability considerations [8, 16, and 22]. Not only the genetic algorithm approach is able to solve the 2383 bus-system problems but also performs better in terms of solution quality as compared to other approaches for solving large scale systems.

TABLE V
PLACEMENT LOCATIONS FOR PMU RELIABILITY OF 0.95

IEEE System	PMU Locations
14	2,4,5,6,7,9,11,13
30	1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 15, 16, 18, 19, 22, 24, 25, 27, 28
57	1, 2, 4, 6, 9, 10, 11, 12, 15, 18, 19, 21, 22, 24, 25, 27, 28, 29, 30, 32, 33, 34, 36, 37, 39, 41, 44, 45, 46, 47, 49, 50, 53, 54, 56
118	1, 2, 5, 6, 7, 9, 10, 11, 12, 15, 17, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 40, 42, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 56, 57, 59, 61, 62, 64, 65, 66, 68, 70, 71, 73, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 89, 90, 91, 92, 94, 96, 100, 101, 105, 106, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118

TABLE VI
PLACEMENT LOCATIONS FOR PMU RELIABILITY OF 0.99

IEEE System	PMU Locations
14	2,6,7,9,13
30	1, 2, 6, 9, 10, 12, 15, 16, 19, 24, 25, 27, 30
57	1, 4, 6, 9, 12, 15, 19, 21, 22, 24, 26, 27, 29, 30, 32, 34, 36, 37, 41, 45, 46, 47, 49, 50, 52, 54, 56
118	1, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 26, 27, 28, 30, 32, 34, 36, 37, 40, 44, 45, 46, 49, 51, 52, 54, 56, 57, 59, 62, 64, 65, 66, 68, 70, 71, 75, 77, 78, 80, 83, 85, 86, 89, 90, 92, 94, 96, 100, 101, 105, 106, 109, 110, 114, 118
2383	All buses except {17, 25, 26, 27, 31, 36, 52, 54, 59, 69, 79, 95, 98, 115, 120, 129, 160, 165, 166, 199, 203, 208, 221, 234, 283, 286, 318, 323, 347, 349, 376, 378, 413, 417, 431, 439, 443, 465, 497, 503, 549, 561, 565, 570, 590, 596, 598, 604, 610, 618, 621, 643, 653, 702, 725, 770, 771, 772, 775, 785, 804, 808, 838, 890, 893, 918, 921, 926, 947, 1055, 1058, 1066, 1088, 1089, 1130, 1143, 1169, 1193, 1196, 1215, 1220, 1223, 1266, 1344, 1372, 1380, 1398, 1411, 1445, 1479, 1500, 1501, 1527, 1536, 1552, 1566, 1579, 1582, 1638, 1658, 1663, 1674, 1702, 1704, 1724, 1742, 1752, 1826, 1833, 1838, 1863, 1881, 1902, 1950, 1960, 1962, 1965, 1971, 2014, 2020, 2037, 2038, 2097, 2138, 2155, 2156, 2194, 2249, 2321, 2344, 2352, 2357, 2380}

By comparing the genetic algorithm (GA) and optimal PMU placement (OPP) results from Table VIII it is clear that although the proposed model requires more PMUs

TABLE VII
PLACEMENT LOCATIONS FOR PMU RELIABILITY OF 0.99833

IEEE System	PMU Locations
14	2,6,7,9
30	1, 2, 6, 9, 10, 12, 15, 19, 25, 27
57	1, 4, 9, 10, 20, 22, 25, 27, 29, 32, 36, 39, 41, 45, 46, 49, 54
118	3, 5, 9, 12, 15, 17, 21, 23, 27, 29, 30, 32, 34, 37, 40, 45, 49, 51, 54, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 89, 92, 96, 100. 105, 110
2383	All buses except {5, 10, 11, 14, 15, 20, 21, 24, 26, 27, 29, 35, 36, 41, 44, 46, 47, 59, 60, 66, 70, 75, 76, 80, 83, 87, 88, 91, 98, 101, 110, 115, 117, 123, 126, 131, 143, 144, 150, 154, 159, 162, 163, 166, 167, 169, 170, 172, 182, 187, 194, 195, 210, 211, 212, 220, 222, 226, 234, 237, 238, 244, 253, 254, 256, 269, 270, 272, 281, 282, 283, 290, 294, 296, 298, 303, 304, 307, 308, 317, 324, 333, 340, 342, 349, 363, 370, 372, 381, 389, 400, 410, 412, 417, 420, 426, 427, 430, 431, 432, 439, 449, 451, 452, 457, 478, 484, 487, 489, 491, 506, 523, 532, 534, 536, 537, 544, 547, 553, 559, 564, 567, 570, 572, 575, 579, 581, 584, 595, 596, 603, 605, 607, 609, 616, 617, 627, 634, 636, 637, 640, 641, 651, 655, 665, 668, 670, 684, 687, 704, 705, 709, 714, 729, 731, 732, 735, 739, 746, 748, 749, 757, 773, 779, 782, 789, 793, 794, 813, 823, 830, 834, 840, 842, 865, 866, 888, 890, 899, 900, 917, 924, 932, 960, 963, 964, 975, 982, 994, 1000, 1001, 1003, 1016, 1021, 1042, 1045, 1062, 1074, 1077, 1079, 1098, 1105, 1117, 1134, 1142, 1144, 1155, 1164, 1169, 1173, 1189, 1194, 1195, 1204, 1207, 1210, 1223, 1227, 1235, 1236, 1252, 1262, 1264, 1276, 1291, 1292, 1312, 1320, 1326, 1328, 1329, 1339, 1342, 1343, 1344, 1363, 1372, 1373, 1374, 1375, 1377, 1390, 1394, 1395, 1401, 1403, 1411, 1417, 1420, 1421, 1427, 1444, 1450, 1459, 1466, 1471, 1478, 1491, 1492, 1495, 1501, 1515, 1517, 1526, 1549, 1553, 1557, 1560, 1563, 1565, 1566, 1567, 1577, 1583, 1586, 1591, 1598, 1606, 1612, 1613, 1634, 1644, 1646, 1650, 1659, 1670, 1683, 1700, 1702, 1705, 1709, 1715, 1718, 1720, 1737, 1743, 1744, 1745, 1752, 1759, 1762, 1775, 1777, 1778, 1788, 1791, 1795, 1801, 1815, 1819, 1831, 1838, 1847, 1848, 1853, 1869, 1891, 1897, 1911, 1916, 1924, 1932, 1935, 1942, 1945, 1947, 1956, 1959, 1960, 1965, 1966, 1967, 1983, 1988, 1989, 1990, 1992, 1995, 2000, 2014, 2039, 2073, 2087, 2088, 2090, 2093, 2097, 2107, 2123, 2126, 2127, 2128, 2136, 2147, 2165, 2171, 2180, 2188, 2193, 2198, 2220, 2227, 2229, 2238, 2242, 2250, 2254, 2264, 2265, 2267, 2285, 2292, 2304, 2309, 2316, 2319, 2322, 2326, 2328, 2331, 2335, 2336, 2341, 2344, 2353, 2359, 2362, 2368, 2375, 2376}

compared to OPP, it achieves a higher system reliability level. Only for the case of IEEE 14 bus system, the OPP performs slightly worse than the GA based solution by placing 4 PMUs and almost reaching minimum required reliability of 0.9 (89.59) as compared to GA based solution. However, in the case of IEEE 118 bus system, system requires 27 additional PMUs to increase the reliability of observability from 44% to desired target value of 90%. It should be noted that in the conventional PMU placement problems, loss of a PMU, would result in loss of the observability of the majority of the neighboring buses. Therefore, loss of a single PMU will result in loss of observability of the system. Hence, such placements are not fault tolerant. Also comparing the GA approach to reliability based placement RBP and GPB, it is clear that the former outperforms the latter. Since the proposed GA model reaches the desired reliability level of 90% approximately 50% lesser number of PMUs that would be required for the RBP based solution, using the GA based approach might lead to significant cost savings. The RBP reaches higher level of reliability of observability, but it might not be required if the target level of reliability is set as 90%.

Hence, the proposed GA model not only is able to solve the large scale problems but also gives a better solution for the majority of the problems as compared to other two approaches by using least number of PMUs given the desired level of reliability. The GA based approach provides the solution with a closer value of system reliability to the target level as compared to other approaches.

In genetic algorithm implementation, the key issues are the representation of the solution as an artificial chromosome and the application of crossover operator. Those choices should be made in accordance with the structure of the problem and solution methodology. Moreover, these two issues have an effect on the scalability of the problem.

The comparison of the effect of PMU reliability on the multi-objective placement has been shown on Fig.2.

TABLE VIII
COMPARISON RESULTS FOR PMU RELIABILITY OF 0.99

	#PMU				R			
IEEE System	GA	GPB	OPP	RBP	GA	GPB	OPP	RBP
14	**5**	**5**	4	9	**0.93**	**0.93**	0.89	0.98
30	**13**	14	10	21	**0.91**	0.93	0.84	0.95
57	**27**	29	17	57	**0.90**	0.92	0.62	0.99
118	**59**	71	32	115	**0.90**	0.91	0.44	0.99
2383	**2250**	N/A	N/A	N/A	**0.90**	N/A	N/A	N/A

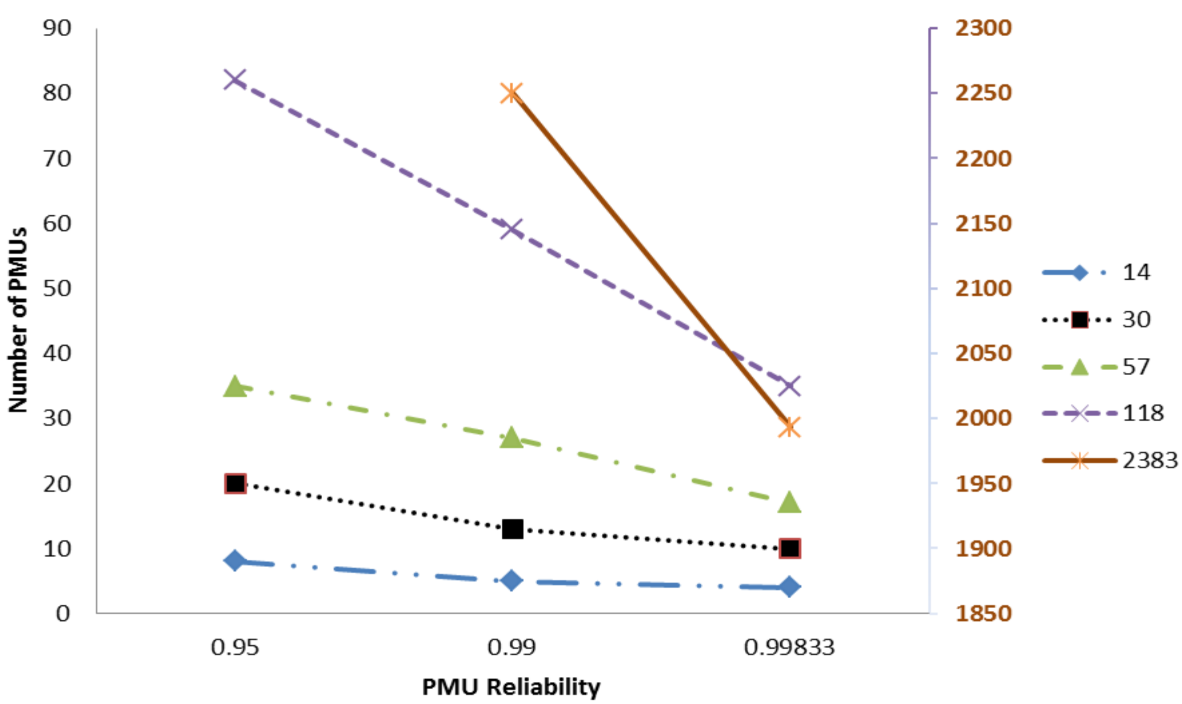

Fig. 2. Effect of PMU reliability on the different IEEE standard test systems.

From the figure, as expected, it can be seen that higher level of individual PMU reliability results in lesser number of PMUs required reaching the desired overall system reliability of observability and the effect increases as the size of the system grows. The secondary y axis indicates the IEEE 2383 bus system values only.
Fig. 3 provides the evolution of the quality of the best solution found during the computation with respect to generations based on individual PMU reliability of 95% for GA. The left y axis indicates number of PMUs, whereas the right y axis indicates the overall score and the total system reliability. The x axis indicates the generation number. Note that throughout the generations, the number of PMUs is decreasing, whereas the overall score that is provided in (4) is increasing. In terms of the total system reliability, there is a fluctuation. Initially, the system reliability at some generations exceeds the level of 0.96, but throughout the generations, it converges to the target level of 0.9, whereas the number of PMU is decreasing initially from the 114 to 82. Throughout better PMU placement, relatively same level of reliability of observability can be retained with less number of PMUs which leads to less costly PMU placement strategies. Another interesting point to note that after approximately generation number of 2750, the population converges and no changes are observed afterwards.

There are other available heuristic procedures, evolutionary procedures that explicitly deal with multiple objectives that could be implemented for this problem. They may be used for benchmark purposes where heuristic approaches including genetic algorithm may be compared based on the quality of the solution and computation time. The required input might be adopted in the form that might be used by the routines in the library.

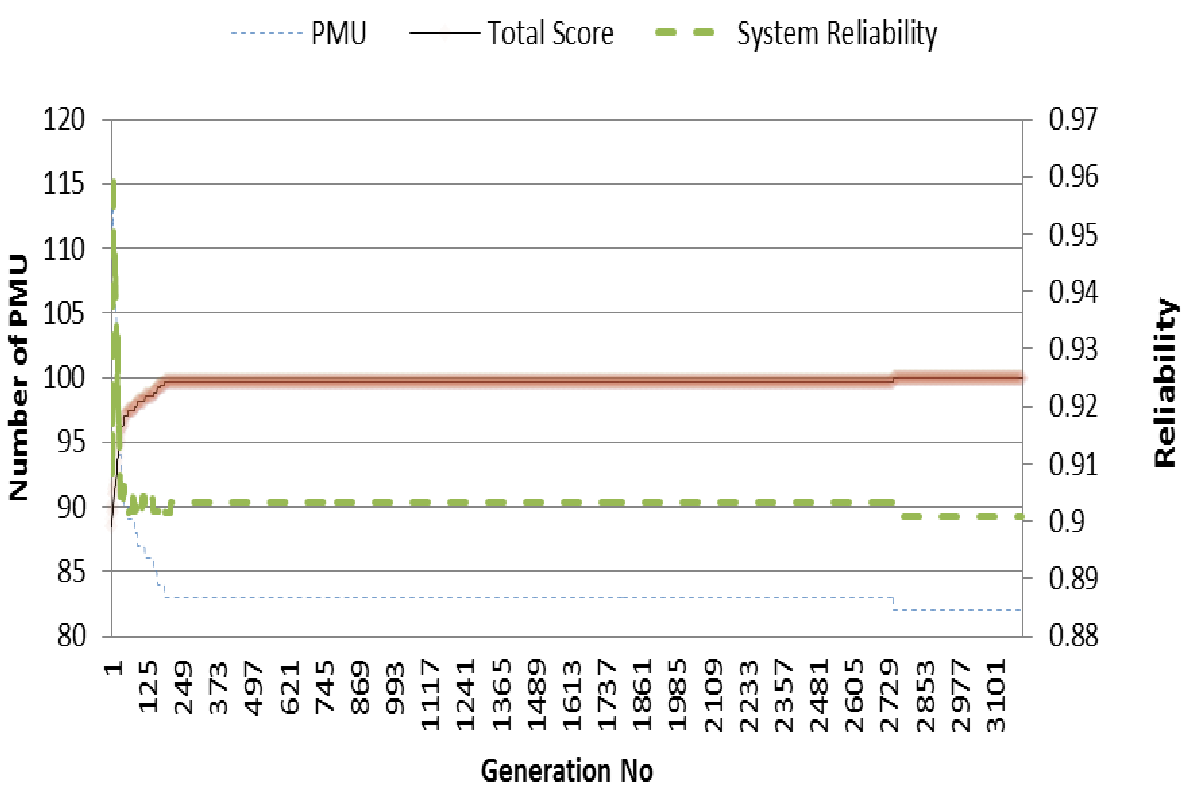

Fig. 3.Evolution of the quality of the incumbent solution provided by the GA for individual PMU reliability of 95% for IEEE 118 bus system.

V. CONCLUSION

In this paper we presented a genetic algorithm based solution methodology to tackle reliability based PMU placement model with two conflicting objective of maximization of the overall system reliability of observability and minimization of the number of PMUs for complete observability of the power network. The proposed genetic algorithm approach is implemented for IEEE 14, 30, 57, 118, and 2383 bus power systems with PMU reliabilities and desired system reliability as inputs.

Comparing to the traditional optimal PMU placement methods, the proposed approach is superior in terms of reliability of system observability. As compared to OPP based approach, where the system reliability of observability is significantly improved from ~45% to more than ~90% for IEEE 118 bus system. However, although the results based on the proposed method require installation of more PMUs, it significantly increases the reliability of observability which is justified based on reliability considerations.

On the other hand, the proposed approach is better as compared to existing reliability based placement approaches. A significant reduction for the number of PMUs installed is achieved under genetic algorithm based approach to reach the minimum desired system reliability of observability of 90% as compared to other approaches (i.e., somewhere between approximately 17-50% for the IEEE 118-bus system).

Moreover, other reliability based approaches fail to provide a solution for IEEE-2383 bus systems, whereas the proposed genetic algorithm provides a solution for this instance at the expense of computation time.

In short, the proposed GA based solution methodology provides a balanced approach for providing the desired level of system reliability of observability with the optimal or near-optimal number of PMUs as compared to other approaches. In a sense, the proposed approach seems to be the most balanced approach where two targets (exceeding the level of the desired reliability of system observability and minimizing the total number of PMUs placed) as compared to reliability based approaches. At the same time, it also considers the reliability perspective of the system that is missed by traditional PMU placement approaches by placing minimum additional number of PMUs at the expense of reduced reliability where the failure of one PMU might result in the total loss of system observability.

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